MADPH-00-1164
hep-ph/0003248
March, 2000

arXiv:hep-ph/0003248v1 24 Mar 2000

EFFECTIVE POTENTIAL CALCULATION OF THE MSSM LIGHTEST CP-EVEN HIGGS BOSON MASS*

REN-JIE ZHANG

Department of Physics, University of Wisconsin, Madison WI 53706 USA

In memory of Prof. Xi-De Xie.

I summarize results of two-loop effective potential calculations of the lightest CP-even Higgs boson mass in the minimal supersymmetric standard model.

Computing the lightest CP-even Higgs boson mass is the most important loop calculation in the minimal supersymmetric standard model because of the paramount importance of a precise m_{h^0} value to the Higgs boson experimental discovery. Tree-level supersymmetry relations require that the Higgs field quartic coupling be related to the electroweak gauge couplings; therefore they impose a strict upper bound $m_{h^0} \leq m_Z$, which is already in conflict with the current lower limit from LEP 2.

It is well-known that this tree-level limit can be drastically changed by radiative corrections. One-loop calculations[1] show that incomplete cancellations of the top and stop loops give positive corrections of the form

$$\Delta m_{h^0}^2 = \frac{3h_t^2 m_t^2}{4\pi^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2} \ , \tag{1}$$

where m_t and $m_{\tilde{t}}$ are top and stop masses respectively. This formula, however, suffers from an ambiguity in the definition of m_t. Numerically, using running or on-shell top-quark mass can amount to about 20% difference in $\Delta m_{h^0}^2$. The problem can only be resolved by an explicit two-loop calculation.

Two-loop calculations in the existing literature have used two different approaches: (a) a renormalization group resummation approach[2], and (b) a two-loop diagrammatic approach[3,4,5]. In the first approach, leading and next-to-leading logarithmic corrections are calculated by integrating one- and two-loop renormalization group equations. However, two-loop non-logarithmic finite corrections are not calculable in principle. The second approach was initiated by Hempfling and Hoang[3] using an effective potential method; they restricted their calculation to specific choice of supersymmetry parameters: *i.e.* large $\tan\beta \to \infty$ and zero left-right stop mixing. Two-loop QCD corrections were later computed at more general cases[5] in the effective potential

*Contribution to PASCOS99: 7th International Symposium on Particles, Strings and Cosmology, Granlibakken, Tahoe City, California, 10-16 Dec 1999.

approach. m_{h^0} to the same two-loop QCD order was also computed[4] using an explicit diagrammatic method. These calculations incorporate both two-loop logarithmic and non-logarithmic finite corrections. In the following, I shall concentrate on the effective potential approach.

The general way of calculating corrections to CP-even Higgs boson mass is to compute Higgs self-energy and tadpole diagrams to the required loop order. In an effective potential approach, these diagrams can be derived from a generating functional, *i.e.* the effective potential, by taking explicit derivatives with respect to the Higgs fields. These quantities then enter the MSSM CP-even Higgs boson mass-squared matrix as follows

$$\mathcal{M}_h^2 = \left[\begin{array}{cc} m_Z^2 c_\beta^2 + m_{A^0}^2 s_\beta^2 + \Delta\mathcal{M}_{11}^2 & -(m_Z^2 + m_{A^0}^2) s_\beta c_\beta + \Delta\mathcal{M}_{12}^2 \\ -(m_Z^2 + m_{A^0}^2) s_\beta c_\beta + \Delta\mathcal{M}_{21}^2 & m_Z^2 s_\beta + m_{A^0}^2 c_\beta + \Delta\mathcal{M}_{22}^2 \end{array} \right] , \quad (2)$$

where $\Delta\mathcal{M}_{ij}^2$ represents radiative corrections to the ij-entry. We note that all these corrections are computed at the zero external momentum limit; sometimes it is necessary to calculate self-energy diagrams directly to account for corrections from non-zero external momenta.

The CP-even Higgs boson masses can be calculated by diagonalizing the above matrix in eq. (2). This computation is tedious but can be greatly simplified when one considers the case $m_{A^0} \gg m_Z$, where m_{A^0} is the mass of the pseudoscalar A^0. In this case, we find the corrections to $m_{h^0}^2$ is

$$\Delta m_{h^0}^2 = \frac{4m_t^4}{v^2} \left(\frac{d}{dm_t^2} \right)^2 V - \mathrm{Re}\ \Pi_{hh}(m_{h^0}^2) + \mathrm{Re}\ \Pi_{hh}(0) \ . \quad (3)$$

where V is the effective potential, v the Higgs field VEV, and the last two terms account for non-zero external momentum corrections.

We have carried out this calculation procedure to the two-loop order including leading QCD[5] and top Yukawa[6] corrections. To illustrate our analysis, we present an approximation formula which is derived under the following assumptions: the soft masses for left and right stops, gluino, heavy Higgs bosons and Higgsinos have a common mass M_S, where M_S can be identified as the supersymmetry scale. The two eigenvalues and mixing angle of stops are then accordingly $m_{\tilde{t}_1}^2 = m_{\tilde{t}}^2 + m_t X_t$, $m_{\tilde{t}_2}^2 = m_{\tilde{t}}^2 - m_t X_t$ and $s_t = c_t = \frac{1}{\sqrt{2}}$, where the average top-squark mass $m_{\tilde{t}}^2 = M_S^2 + m_t^2$, and $X_t = A_t + \mu/\tan\beta$ is the left-right stop mixing parameter.

We find the approximation formula for two-loop QCD+top Yukawa corrections is[6] (in terms of on-shell mass parameters)

$$\Delta m_{h^0}^2 = \frac{3m_t^4}{2\pi^2 v^2} \left(\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \hat{X}_t^2 - \frac{\hat{X}_t^4}{12} \right)$$

$$
\begin{aligned}
&+\frac{\alpha_s m_t^4}{\pi^3 v^2}\left(-3\ln^2\frac{m_{\tilde t}^2}{m_t^2}-6\ln\frac{m_{\tilde t}^2}{m_t^2}+6\hat X_t-3\hat X_t^2\ln\frac{m_{\tilde t}^2}{m_t^2}-\frac{3\hat X_t^4}{4}\right)\\
&+\frac{3\alpha_t m_t^4}{16\pi^3 v^2}\Bigg\{s_\beta^2\left(3\ln^2\frac{M_S^2}{m_t^2}+13\ln\frac{M_S^2}{m_t^2}\right)-1-\frac{\pi^2}{3}+c_\beta^2\left(60K+\frac{13}{2}+\frac{4\pi^2}{3}\right)\\
&+\left[3s_\beta^2\ln\frac{M_S^2}{m_t^2}-c_\beta^2\left(\frac{69}{2}+24K\right)+41\right]\hat X_t^2-\left(1+\frac{61}{12}s_\beta^2\right)\hat X_t^4+\frac{s_\beta^2}{2}\hat X_t^6\\
&+c_\beta^2\Bigg[(3-16K-\sqrt{3}\pi)(4\hat X_t\hat Y_t+\hat Y_t^2)+\left(16K+\frac{2\pi}{\sqrt{3}}\right)\hat X_t^3\hat Y_t\\
&+\left(-\frac{4}{3}+24K+\sqrt{3}\pi\right)\hat X_t^2\hat Y_t^2-\left(\frac{7}{12}+8K+\frac{\pi}{2\sqrt{3}}\right)\hat X_t^4\hat Y_t^2\Bigg]\Bigg\}\,,
\end{aligned}
\tag{4}
$$

where the constant $K \simeq -0.195$. We note that two-loop QCD corrections depend only on $\hat X_t = X_t/m_{\tilde t}$ while the top Yukawa corrections depend on $\hat Y_t = (A_t - \mu\tan\beta)/m_{\tilde t}$ as well. This approximation formula is good to a level of 0.5 GeV for most of the parameter space.

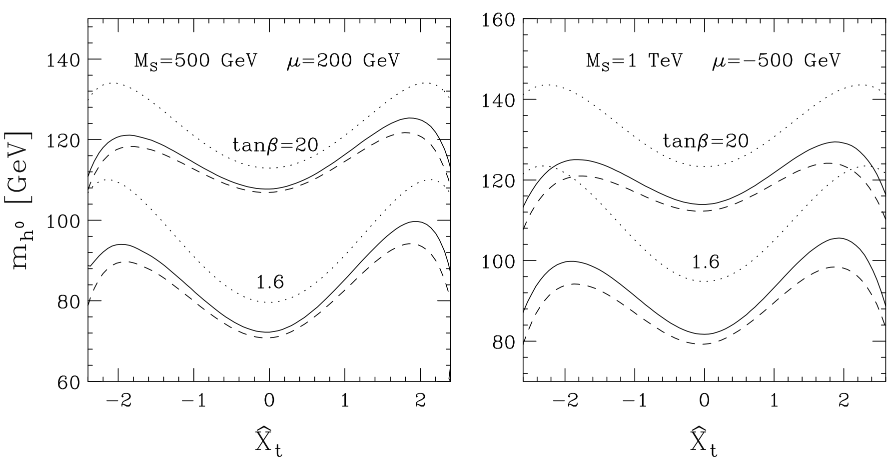

Figure 1. Higgs boson mass m_{h^0} versus $\hat X_t$. The dotted, dot-dashed and solid lines correspond to Higgs boson masses calculated to the orders of one-loop, two-loop QCD and two-loop QCD+top Yukawa respectively.

Fig. 1 shows the Higgs boson mass m_{h^0} vs. the stop mixing parameter $\hat X_t$, for different choices of M_S, μ and $\tan\beta$. The two-loop QCD corrections agree well with other approaches[4]. They generally decrease m_{h^0} from their

one-loop values by $10 - 20$ GeV depending on the parameter choice. Two-loop Yukawa corrections are sizeable for large stop mixings, in particular, for $\hat{X}_t \simeq \pm 2$ two-loop Yukawa corrections can increase m_{h^0} by about 5 GeV.

Another interesting feature observed in the literature[4,5] is that two-loop corrections shift the maximal mixing peaks. At the one-loop level, these peaks are at $\hat{X}_t = \pm\sqrt{6}$. It is easy to see from eq. (4) that the size of shifts is about 10%, *i.e.* the peaks move to $\hat{X}_t \simeq \pm 2$. This is confirmed by Fig. 1.

Finally, renormalization group resummation technique can be used to derive a particularly nice mass correction formula which has clearer physical interpretations. We find eq. (4) can be transformed into the following form by using solutions to the renormalization group equations

$$\Delta m_{h^0}^2 = \frac{3\overline{m}_t^4(Q_t)}{2\pi^2\overline{v}^2(Q_1^*)} \ln \frac{m_{\tilde{t}}^2(Q_{\rm th})}{\overline{m}_t^2(Q_t')} + \frac{3\overline{m}_t^4(Q_{\rm th})}{2\pi^2\overline{v}^2(Q_2^*)} \left[\hat{X}_t^2(Q_{\rm th}) - \frac{\hat{X}_t^4(Q_{\rm th})}{12} \right] + \Delta_{\rm th}^{(2)} , \tag{5}$$

where $Q_1^* = e^{-1/3} m_t$, $Q_2^* = e^{1/3} m_t$, $Q_t = \sqrt{m_t m_{\tilde{t}}}$, $Q_t' = (m_t m_{\tilde{t}}^2)^{1/3}$ and $Q_{\rm th} = m_{\tilde{t}}$, $\overline{v}$ and $\overline{m}$ are the Standard Model $\overline{\rm MS}$ parameters. These choices of scales for evaluating one-loop corrections automatically take into account two-loop leading and next-to-leading logarithmic effects. The leftover finite correction term $\Delta_{\rm th}^{(2)}$ is understood as two-loop threshold corrections and numerically small; its detail form can be found in a forthcoming paper[6].

I thank J. R. Espinosa for collaborations. This work was supported in part by a DOE grant No. DE-FG02-95ER40896 and in part by the Wisconsin Alumni Research Foundation.

References

1. Y. Okada, M. Yamaguchi, and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1; J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B257** (1991) 83; H.E. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815.
2. M. Carena, J.R. Espinosa, M. Quirós and C.E.M. Wagner, *Phys. Lett.* **B355** (1995) 209; M. Carena, M. Quirós and C.E.M. Wagner, *Nucl. Phys.* **B461** (1996) 407; H.E. Haber, R. Hempfling and A.H. Hoang, *Z. Phys.* **C75** (1997) 539.
3. R. Hempfling and A.H. Hoang, *Phys. Lett.* **B331** (1994) 99.
4. S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev.* **D58** (1998) 091701; *Eur. Phys. J.* **C9** (1999) 343.
5. R.-J. Zhang, *Phys. Lett.* **B447** (1999) 89; J.R. Espinosa and R.-J. Zhang, hep-ph/9912236.
6. J.R. Espinosa and R.-J. Zhang, hep-ph/0003246.